

№ 6.5

$$V_{gr} = \frac{\partial \omega}{\partial k} \Rightarrow V_{gr}^{-1} = \frac{\partial k}{\partial \omega}, \quad V_{ph} = \frac{c_0}{n}, \quad V_{ph}^{-1} = \frac{n}{c_0} = \frac{k}{\omega}$$

$$V_{gr}^{-1} = \frac{\partial}{\partial \omega} \left(\frac{n(\omega) \cdot \omega}{c_0} \right) = \frac{n}{c_0} + \frac{\omega}{c_0} \cdot \frac{\partial n}{\partial \omega}$$

$$V_{gr}^{-1} - V_{ph}^{-1} = \frac{n}{c_0} - \frac{\omega}{c_0} \frac{\partial n}{\partial \omega} - \frac{n}{c_0} = -\frac{\omega}{c_0} \frac{\partial n}{\partial \omega} = -\frac{2\pi}{\lambda} \frac{\partial n}{\partial \lambda} \frac{d\lambda}{d\omega} \quad \textcircled{=}$$

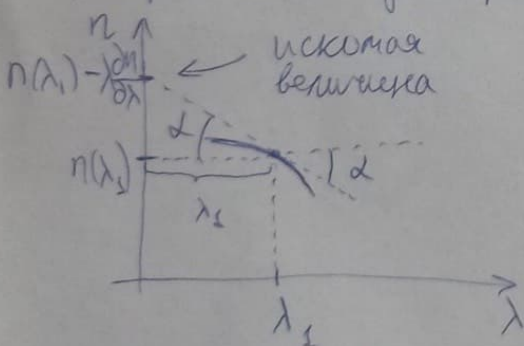
$$\lambda = \frac{2\pi c_0}{\omega}, \quad \omega = \frac{2\pi c_0}{\lambda}$$

$$\textcircled{=} \frac{2\pi}{\lambda} \cdot \left(-\frac{2\pi c_0}{\omega^2} \right) \frac{\partial n}{\partial \lambda} = -\frac{c_0}{\lambda} \left(\frac{\lambda}{c_0} \right)^2 \frac{\partial n}{\partial \lambda} = -\frac{\lambda}{c_0} \frac{\partial n}{\partial \lambda}$$

$$\Rightarrow \frac{1}{V_{gr}} = \frac{1}{V_{ph}} - \frac{\lambda}{c_0} \frac{\partial n}{\partial \lambda}$$

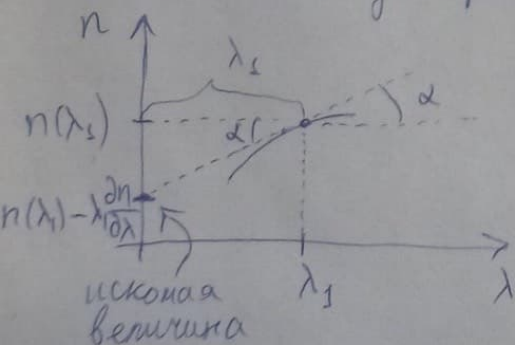
$$\frac{c_0}{V_{gr}} = \frac{c_0}{V_{ph}} - \frac{\lambda \cdot c_0}{c_0} \frac{\partial n}{\partial \lambda} = \boxed{n - \lambda \frac{\partial n}{\partial \lambda}}$$

1) нормальная дисперсия: $\frac{\partial V_{ph}}{\partial \lambda} > 0$. $\frac{\partial V_{ph}}{\partial \lambda} = -\frac{c_0}{n^2} \frac{\partial n}{\partial \lambda} \Rightarrow \frac{\partial n}{\partial \lambda} < 0$

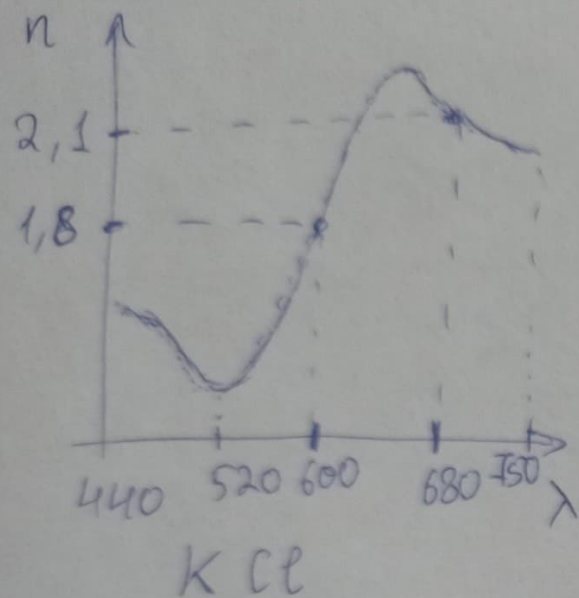


$$\tan \alpha = \frac{\partial n}{\partial \lambda} < 0$$

2) аномальная дисперсия: $\frac{\partial V_{ph}}{\partial \lambda} < 0 \Rightarrow \frac{\partial n}{\partial \lambda} > 0$



$$\tan \alpha = \frac{\partial n}{\partial \lambda} > 0$$



$$\lambda_1 = 600 \text{ нм}$$

$$\lambda_2 = 680 \text{ нм}$$

$$\frac{\partial n(\lambda_1)}{\partial \lambda} \approx \frac{1,1}{80 \text{ нм}} > 0 - \text{аном. дисп.}$$

$$\frac{\partial n(\lambda_2)}{\partial \lambda} \approx -\frac{0,4}{120 \text{ нм}} < 0 - \text{норм. дисп.}$$

$$n(\lambda_1) \approx 1,8$$

$$n(\lambda_2) \approx 2,1$$

$$\frac{c}{v_{\text{гр}}} \Big|_{\lambda=\lambda_1} = 2,1 - 1,8 + \frac{0,4}{120 \text{ нм}} \cdot 680 \text{ нм} \approx 4,4$$

$$\frac{c}{v_{\text{гр}}} \Big|_{\lambda=\lambda_2} = 2,1 + \frac{0,4}{120 \text{ нм}} \cdot 680 \text{ нм} \approx 4,4$$

$$\frac{c}{v_{\text{гр}}} \Big|_{\lambda=\lambda_1} = 1,8 - \frac{1,1}{80 \text{ нм}} \cdot 600 \text{ нм} \approx -6,45$$